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Analysis of the Impact of Kinematic Parameters on Kalman Filtering Performance in Missile Angular Tracking Systems

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Abstract:

In navigation and target tracking control systems, accurate angular tracking is a crucial requirement to ensure system stability and precision. However, measurement noise significantly degrades signal quality and impacts target tracking capabilities. This paper presents a simulation model of an angle tracking system using a discrete Kalman filter, built upon the differential equations of an electromechanical rotating system. The novelty of this study lies in evaluating the impact of three characteristic kinematic parameters moment of inertia, damping coefficient, and stiffness on filtering performance and tracking error. By calculating and visualizing RMSE, velocity error, and angular error over time, this study clarifies the design parameters' role in improving angle signal filtering quality. The simulation results are visualized and serve as a reference for selecting optimal parameter configurations in real-world tracking systems under noise conditions.

Keywords: *Angle tracking system, Kalman filter, angular error, simulation, RMSE, kinematic parameters.*

I. INTRODUCTION

In observation, navigation, or fire control systems, tracking the angular motion of a target is essential and directly affects the system's accuracy and combat effectiveness. In real-world conditions, sensor measurements are often corrupted by noise, resulting in significant deviations in measured angles from the true values. Kalman filters are widely used to address this issue due to their ability to combine system models with measured signals for better state estimation. Additionally, the dynamics of angle tracking systems depend on design-specific parameters such as the moment of inertia, damping coefficient, and stiffness. However, the impact of these parameters on filtering quality under stochastic noise conditions remains underexplored. This paper focuses on simulating an angle tracking system with integrated Kalman filtering using three different sets of kinematic parameters and evaluates filtering performance through standard error indices. This is a crucial step toward determining optimal system configurations for noisy environments.

II. ALGORITHM SYNTHESIS AND SYSTEM MODELING

This section presents the theoretical foundation of the angle tracking model and the discrete Kalman filter algorithm used in the simulation program. The equations are developed based on the kinematics of a rotating servo system and then discretized for numerical implementation.

The angle tracking system is commonly modeled as an electromechanical rotational system influenced by control and resistive torques. The rotational motion equation is:

$$J\frac{d^2\theta}{dt^2}+D\frac{d\theta}{dt}+K\theta=K\theta_c \quad (1)$$

Where:

$\theta(t)$ - Actual rotation angle of the tracking system [deg];

$\theta_c(t)$ - Command angle [deg];

J - Moment of inertia of the tracker [kg.m^2];

D - Rotational damping coefficient [N.m.s/deg];

K - Torsional stiffness coefficient [N.m/deg].

Equation (1) can be normalized to:

$$\frac{d^2\theta}{dt^2}+\frac{D}{J}\frac{d\theta}{dt}+\frac{K}{J}\theta=\frac{K}{J}\theta_c \tag{2}$$

To apply the Kalman filter, the system must be expressed in state-space form. Let:

$$x(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}=\begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix}\Rightarrow \dot{x}(t)=\begin{bmatrix} \dot{\theta}(t) \\ \ddot{\theta}(t) \end{bmatrix} \tag{3}$$

Then the state-space equation becomes:

$$\dot{x}(t)=Ax(t)+Bu(t), y(t)=Cx(t) \tag{4}$$

Where:

$u(t)=\theta_c(t)$ - Input to the angle tracking system [deg];

$y(t)=\theta(t)$ - Output of the angle tracking system [deg].

The matrices are defined as:

$$A=\begin{bmatrix} 0 & 1 \\ -\frac{K}{J} & -\frac{D}{J} \end{bmatrix}, B=\begin{bmatrix} 0 \\ \frac{K}{J} \end{bmatrix}, C=\begin{bmatrix} 1 & 0 \end{bmatrix} \tag{5}$$

Since the simulation runs on a computer, the system is discretized using a time step T [s]. Using the Euler method:

$$\begin{aligned} & x(k+1)\approx x(k)+T.\dot{x}(k) \\ & \Rightarrow A_d=I+A.T, B_d=B.T \end{aligned} \tag{6}$$

Where:

A_d - Discrete state matrix;

B_d - Discrete control matrix.

The discrete noisy system is represented by the Kalman filter model:

$$x_{k+1}=Ax_k+Bu_k+\omega_k, z_k=Cx_k+\upsilon_k \tag{7}$$

Where:

x_k - True state at time step k;

z_k - Noisy measurement [deg];

$\omega_k \sim \mathrm{N}(0,Q)$ - Process noise [deg^2];

$\upsilon_k \sim \mathrm{N}(0,R)$ - Measurement noise [deg^2];

Q - Process noise covariance matrix, typically small;

R - Measurement noise covariance matrix, defined by sensor.

Kalman Filter Algorithm Steps:

Step 1: State Prediction

$$\begin{aligned} & \hat{x}_{k/k-1}=A\hat{x}_k+Bu_{k-1} \\ & P_{k/k-1}=AP_{k-1}A^T+Q \end{aligned} \tag{8}$$

Where:

$\hat{x}_{k/k-1}$ - Predicted state (prior to update);

$P_{k/k-1}$ - Predicted error covariance [deg^2].

Step 2: Kalman Gain Calculation

$$K_k=P_{k/k-1}C^T CP_{k/k-1}C^T+R^{-1} \tag{9}$$

K_k - Kalman gain determines the weight between model prediction and actual measurement.

Step 3: State and Covariance Update

$$\begin{aligned} & \hat{x}_k=\hat{x}_{k/k-1}+K_k(z_k-C\hat{x}_{k/k-1}) \\ & P_k=(I-K_kC)P_{k/k-1} \end{aligned} \tag{10}$$

Here, the state is corrected based on the difference between the actual and predicted measurement. After filtering, results are evaluated using:

Root Mean Square Error (RMSE):

$$RMSE=\sqrt{\frac{1}{N}\sum_{k=1}^{N}(x_k-\hat{x}_k)^2} \tag{11}$$

Bias (Mean Error):

$$Bias=\frac{1}{N}\sum_{k=1}^{N}(x_k-\hat{x}_k)^2 \tag{12}$$

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Standard Deviation of Error:

$$\sigma = \sqrt{\frac{1}{N}\sum_{k=1}^{N}[(x_k - \hat{x}_k) - Bias]^2} \quad (13)$$

III. SIMULATION AND EVALUATION

To assess the effectiveness of the Kalman filter in angle tracking systems and examine how kinematic parameters (inertia *J*, damping *D*, stiffness *K*) influence filtering error, the authors built a simulation with three representative parameter sets reflecting different dynamic characteristics of electromechanical systems. The input signal is a time-varying step function, and Gaussian noise is added to simulate real-world conditions. Comparing RMSE, bias, and standard deviation in each case allows for quantitative evaluation of Kalman filter performance with each parameter set.

- Set 1: J = 0.01 (kg.m²), D = 0.05 (N.m.s/deg), K = 1.0 (N.m/deg);

- Set 2: J = 0.02 (kg.m²), D = 0.10 (N.m.s/deg), K = 0.8 (N.m/deg);

- Set 3: J = 0.015 (kg.m²), D = 0.08 (N.m.s/deg), K = 0.9 (N.m/deg).

Gaussian measurement noise with a standard deviation of σ = 0.5 (deg) is added. The discrete Kalman filter is implemented with Q = 0.001.I and R = σ². The simulation results are as follows:

Figure 1. Kalman filter output compared to true angle and noisy measurement

Figure 1 illustrates the angular tracking performance of the system under Gaussian measurement noise conditions. The black line denotes the true target angle, while the red dotted line represents the measured signal corrupted by noise (standard deviation σ = 0.5 (deg)). After applying the discrete Kalman filter with three different sets of dynamic parameters, all filtered outputs closely track the true angle. Parameter Set 1 yields the fastest response but exhibits slight overshoot and oscillations near the switching times at 5 and 10 seconds. Parameter Set 2 responds more slowly, effectively reducing oscillations but introducing noticeable delay during transients. Parameter Set 3 achieves a balanced trade-off between tracking accuracy and response smoothness. These results highlight the critical influence of dynamic parameters on filtering performance and transient characteristics.

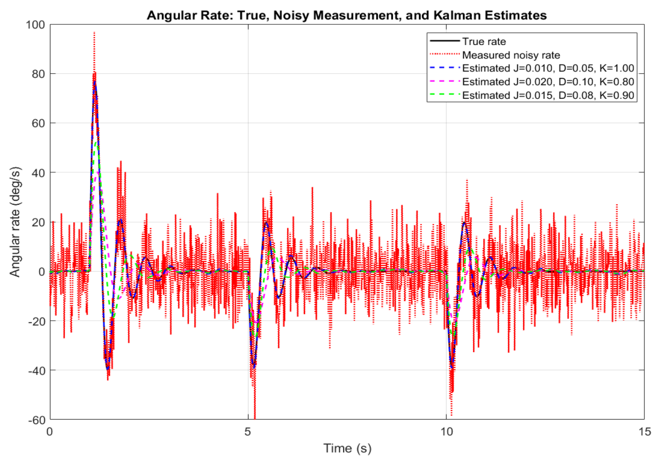

Figure 2. Kalman filter output compared to true angular rate and noisy measurement

Figure 2 illustrates the angular rate response of the system, including the true angular velocity, the noisy measured rate obtained from differentiated angle measurements, and the Kalman filter estimates for three different parameter configurations. As expected, the measured angular rate exhibits significant high-frequency noise due to the differentiation of noisy angle measurements, leading to large amplitude fluctuations that do not reflect the true system dynamics. In contrast, the Kalman filter effectively suppresses measurement noise while preserving the transient and steady-state characteristics of the angular rate. The estimated angular rates closely follow the true response during both transient events and steady-state intervals, demonstrating the robustness of the Kalman filter against measurement noise and parameter variations. Minor discrepancies between the estimates are observed during rapid transients, which can be attributed to differences in inertia, damping, and stiffness parameters. Overall, the results confirm the effectiveness of the Kalman filtering approach in reconstructing angular velocity from noisy angle measurements.

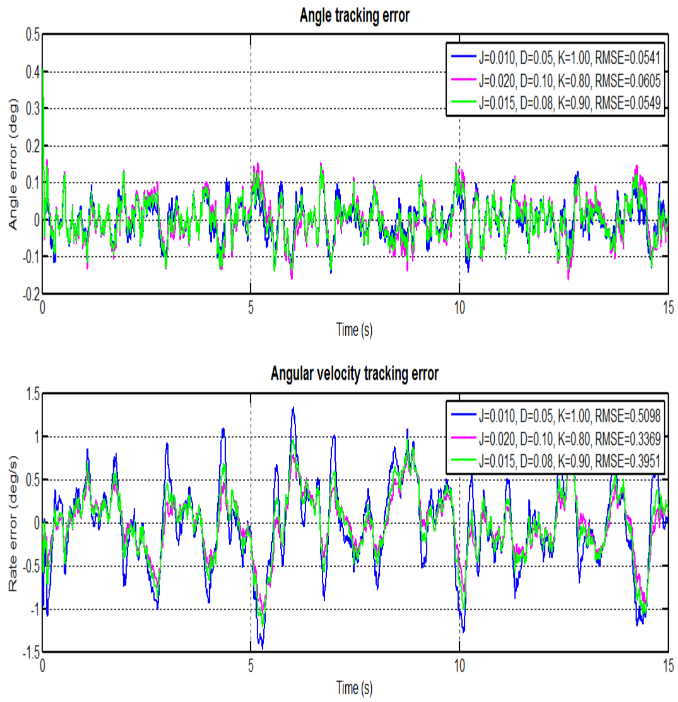

Figure 3. Angular error and angular velocity error for each parameter set

Figure 3 shows the estimation errors in angle (top plot) and angular velocity (bottom plot) across the three parameter configurations. The angular error generally remains within ±0.1 (deg), and the low RMSE values indicate the effectiveness of the Kalman filter in angle estimation. Specifically, Parameter Set 1 yields an RMSE of 0.0541 (deg), Set 2 results in 0.0605 (deg), and Set 3 achieves 0.0549 (deg), suggesting minimal angular estimation error and weak dependence on system inertia. However, velocity error reveals more significant differences: Set 1 exhibits the largest error (RMSE = 0.5098deg/s) due to its faster and more oscillatory response; Set 2 achieves the smallest error (RMSE = 0.3369deg/s), benefiting from higher inertia and damping; while Set 3 provides intermediate performance (RMSE = 0.3951deg/s). These results demonstrate that appropriate selection of system dynamics can balance between positional accuracy and responsiveness in angular tracking systems.

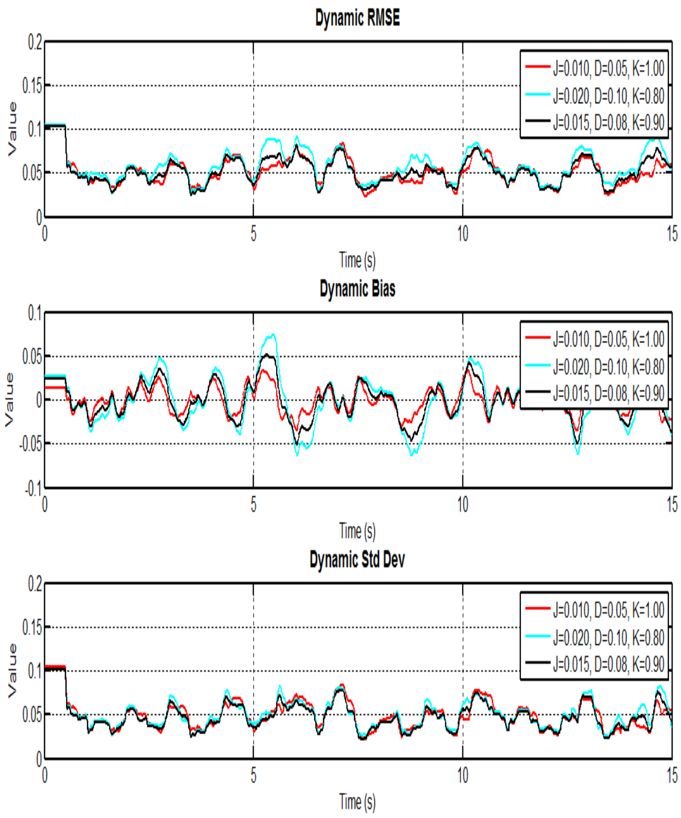

Figure 4. RMSE, bias, and standard deviation over time

Figure 4 presents the temporal evolution of estimation errors using a sliding window of 50 samples. The top graph shows the dynamic RMSE, where Parameter Set 2 tends to have higher values, particularly near the step transitions at 5 and 10 seconds indicative of slower adaptation due to greater inertia. The middle graph shows the bias remaining close to zero for all configurations, ensuring unbiased estimates; however, Set 2 exhibits a noticeable positive bias in the 4–6 second interval. The bottom graph displays the standard deviation of estimation error: Sets 1 and 3 remain within the 0.03–0.07 (deg) range, while Set 2 peaks at approximately 0.09 (deg), indicating that reduced oscillation in the response does not necessarily equate to lower residual noise. These findings emphasize the need for time-resolved error analysis to select optimal dynamic parameters tailored to different operating phases of the tracking system.

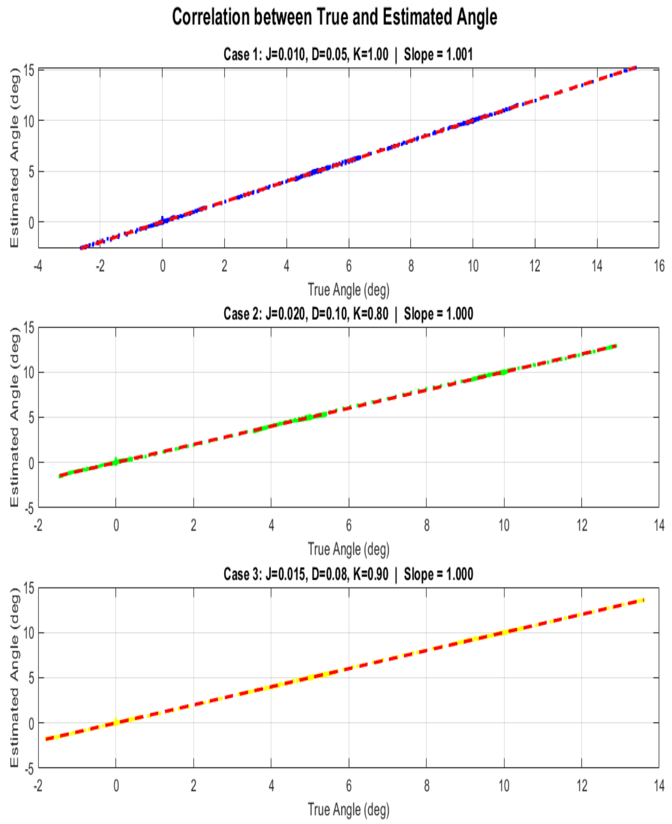

Figure 5. Correlation between true angle and Kalman filter estimated angle for different system parameter sets

Figure 5 illustrates the correlation between the true angle and the angle estimated by the Kalman filter for three different system parameter sets. In all cases, the data points are tightly clustered along the fitted linear regression line with a slope close to unity, indicating high estimation accuracy and negligible bias.

For the case with smaller inertia (J = 0.010) and higher stiffness (K = 1.00), the estimator maintains excellent linearity, demonstrating the ability of the Kalman filter to accurately track rapid system dynamics. When the inertia and damping are increased (J = 0.020, D = 0.10), the slope of the regression line remains close to one, reflecting the robustness and stability of the filter under parameter variations. The intermediate parameter set (J = 0.015, D = 0.08, K = 0.90) exhibits similar behavior, further confirming that the Kalman filter performance is weakly sensitive to changes in system parameters within the considered range. Overall, these results demonstrate that the Kalman filter provides reliable and nearly unbiased angle estimates across different system configurations.

IV. CONCLUSION

This paper has presented a comprehensive simulation-based analysis of a Kalman-filtered angular tracking system, with particular emphasis on the influence of kinematic parameters, including moment of inertia, damping coefficient, and stiffness, on filtering performance under noisy measurement conditions. Through the evaluation of angle response, angular rate estimation, tracking errors, dynamic RMSE, and correlation characteristics, the results demonstrate that the Kalman filter is capable of providing accurate and nearly unbiased angle and angular rate estimates across a wide range of system parameter configurations. Systems with lower inertia and higher stiffness exhibit faster responses but tend to introduce larger transient oscillations, while increased inertia and damping improve noise suppression at the expense of response speed. The correlation analysis further confirms the robustness of the Kalman filter, with estimated angles showing a strong linear relationship with true values and regression slopes close to unity in all cases. Overall, the findings highlight that appropriate selection of kinematic parameters plays a crucial role in balancing tracking accuracy, noise attenuation, and dynamic responsiveness. The proposed simulation framework and analysis provide a valuable reference for the design and tuning of Kalman-filter-based angular tracking systems in missile guidance and other high-dynamic applications operating under stochastic noise environments.

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