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Synthesis of Optimal Guidance Laws for Surface-to-Air Missiles in the Remote Control Phase Based on the Standard Trajectory Method

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Abstract:

This paper presents a method for synthesizing optimal guidance laws for surface-to-air missiles during the remote control phase based on the standard trajectory method. In the initial stage of missile flight, remote control is crucial to ensure accurate transition to the homing phase and effective interception of maneuvering aerial targets. By establishing a standard reference trajectory and applying optimal control theory, the proposed approach minimizes trajectory deviations and control energy while maintaining high terminal accuracy. The standard trajectory method enables analytical determination of control commands that guarantee stable flight and rapid convergence toward the desired path. Simulation results demonstrate that the synthesized guidance law improves control precision, reduces response delay, and enhances overall missile–target engagement performance compared with conventional proportional navigation and linear feedback laws. The proposed method can be effectively applied to modern SAM systems to improve interception probability in complex air defense scenarios.

Keywords— Guidance laws, Surface-to-air missiles, Control phase

I. INTRODUCTION

The remote control (or command guidance) phase of surface-to-air missiles (SAMs) plays a decisive role in shaping the missile's flight trajectory prior to the transition to terminal homing. During this stage, accurate command generation ensures that the missile maintains favorable geometry and energy conditions for successful target interception, particularly when engaging high-speed or maneuvering aerial threats [1]. Many modern SAM systems, such as command-to-line-of-sight (CLOS) or track-via-missile architectures, still rely on remote control to manage the boost and midcourse phases before the seeker is activated [2].

Traditional guidance laws, notably proportional navigation (PN), have been widely applied due to their simplicity, low computational cost, and robustness in diverse scenarios [1], [3]. However, as modern air-combat environments increasingly involve highly maneuverable targets, electronic countermeasures, and strict energy and control constraints, the classical PN law often becomes suboptimal. Consequently, advanced control methodologies-including optimal control, linear quadratic regulation (LQR), and model predictive control (MPC) - have been proposed to improve guidance precision and energy efficiency while maintaining robustness to system and target uncertainties [3], [4]. The standard trajectory method introduces a nominal or reference trajectory that represents the desired missile path under standard engagement conditions. By using this standard trajectory as a reference, the missile's remote control problem can be reformulated as an optimal tracking control problem, minimizing deviations from the reference while conserving control effort [4], [5]. This approach allows the system to achieve smoother control commands, enhanced stability, and better terminal conditions for the homing phase, especially when dynamic constraints and real-time computation limits are considered. Despite the method's practical advantages, there remains a lack of research integrating the standard trajectory concept with modern optimal control synthesis in the remote control phase of SAMs. Existing studies have mainly focused on the design of optimal homing guidance or simplified midcourse shaping with linear dynamics [3]–[5]. Therefore, developing a comprehensive optimal guidance synthesis framework that explicitly incorporates a standard trajectory during the remote control phase is both technically important and practically relevant. This paper aims to formulate and solve the optimal guidance law for SAMs during the remote control phase using the standard trajectory method. The proposed scheme minimizes trajectory deviation, terminal error, and control energy simultaneously. Analytical derivations and simulation results demonstrate that the synthesized law achieves higher trajectory accuracy and energy efficiency compared with classical PN-based command guidance.

II. SYNTHESIS OF OPTIMAL GUIDANCE LAWS FOR SURFACE-TO-AIR MISSILES BASED ON THE STANDARD TRAJECTORY METHOD

The motion of the rocket in the horizontal plane of the launch coordinate system XOZ is shown in figure 1 below.

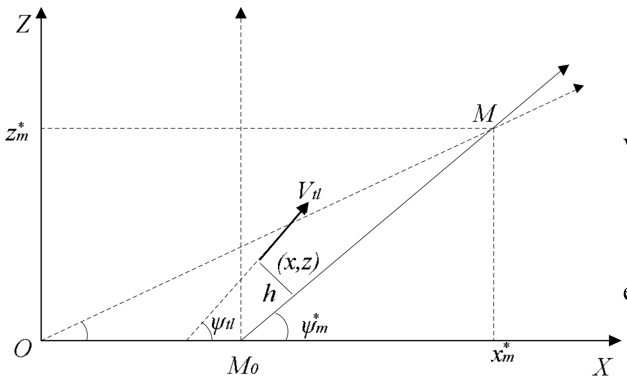

Figure 1 Missile position relative to the reference trajectory.

Considering the motion of the missile in the plane M_0xz, the kinematic equation of the missle is given by:

$$\begin{cases} \Delta\dot{\psi} = \frac{g}{V_{tl}} n_z \\ \dot{x}_1 = V_{tl}\cos\Delta\psi \\ \dot{z}_1 = V_{tl}\sin\Delta\psi \end{cases} \quad (1)$$

Notation: (x_m, z_m), (x_1, z_1), are the reference trajectory coordinates and missile coordinates at the time t in the coordinate system, respectively. The deviation between the missile trajectory and the reference trajectory is determined as follows:

$$\Delta h = h - z_m = z_1 - z_m$$

Derivative of both sides we have:

$$\Delta\dot{h} = \dot{z}_1 - \dot{z}_m = \dot{z}_1 - \frac{\partial f(x_1)}{\partial x_1}\dot{x}_1,$$

symbol $f_x = \frac{\partial f(x_1)}{\partial x_1}$, replace $\dot{x}_1$, $\dot{z}_1$ from system (1) to get: $\Delta\dot{h} = V_{tl}\sin\Delta\psi - f_x V_{tl}\cos\Delta\psi$

$$\Leftrightarrow \Delta\dot{h} = V_{tl}\sqrt{1+f_x^2}\left(\frac{1}{\sqrt{1+f_x^2}}\sin\Delta\psi - \frac{f_x}{\sqrt{1+f_x^2}}\cos\Delta\psi\right)$$

In there:

$$\sin\psi_m = \frac{f_x}{\sqrt{1+f_x^2}};\ \cos\psi_m = \frac{1}{\sqrt{1+f_x^2}};\ \tan\psi_m = f_x.$$

We have, ψ_m is the reference orbital tilt angle at time t. Let: $\Delta\bar{\psi} = \Delta\psi - \psi_m$, with $\Delta\psi$ is the deviation of the missile orbital angle from the reference orbital tilt angle. The derivative of both sides is obtained:

$$\Delta\dot{\bar{\psi}} = \Delta\dot{\psi} - \dot{\psi}_m \quad (3)$$

Differentiate both sides and $\tan\psi_m = f_x$ we get:

$$\frac{1}{\cos^2\psi_m}\dot{\psi}_m = f_{xx}\dot{x}_1 = f_{xx}V_{tl}\cos\Delta\psi,$$

$$\Leftrightarrow \quad \dot{\psi}_m = f_{xx}\cos^2\psi_m V_t\cos\Delta\psi \quad (4)$$

With:

$$f_{xx} = \frac{\partial^2 f(x_1)}{\partial x_1^2}$$

Substitute (1) into (2) and (3) combined with the first equation of system (4) we get:

$$\begin{cases} \Delta\dot{\bar{\psi}} = \frac{g}{V_{tl}} n_z - \dot{\psi}_m \\ \Delta\dot{h} = V_{tl}\sqrt{1+f_x^2}\sin(\Delta\bar{\psi}) \end{cases} \quad (5)$$

Put $n_{zm} = \sqrt{1+f_x^2}\left(n_z - \frac{V_{tl}}{g}\dot{\psi}_m\right)$; $\bar{V}_{tl} = V_{tl}\sqrt{1+f_x^2}$

System (5) becomes:

$$\begin{cases} \Delta\dot{\bar{\psi}} = \frac{g}{\bar{V}_{tl}} n_{zm} \\ \Delta\dot{h} = \bar{V}_{tl}\sin(\Delta\bar{\psi}) \end{cases} \quad (6)$$

Boundary conditions at $t_f: \Delta\bar{\psi}_{t_f} \to 0;\ \Delta h_{t_f} \to 0$ with $t_f < t_{td}$.

The quality index function has the form:

$$J = \frac{1}{2}\rho_1\Delta\bar{\psi}_{t_f}^2 + \frac{1}{2}\rho_2\Delta h_{t_f}^2 + \frac{1}{2}\int_{t_o}^{t_f} n_{zm}^2 dt \to \min \quad (7)$$

From (7) we get the Terminant function:

$$G = \frac{1}{2}\begin{bmatrix}\Delta\bar{\psi} \\ \Delta h\end{bmatrix}_{t_f}^T \left[P^f\right]_{2\times 2}\begin{bmatrix}\Delta\bar{\psi} \\ \Delta h\end{bmatrix}_{t_f};$$

$$P^f = \begin{bmatrix}\rho_1 & 0 \\ 0 & \rho_2\end{bmatrix}; \quad (8)$$

Hamiltonian function:

$$H = \lambda_{\Delta\bar{\psi}}\frac{g}{\bar{V}_{tl}}n_{zm} + \lambda_{\Delta h}\bar{V}_{tl}\sin\Delta\bar{\psi} + \frac{1}{2}n_{zm}^2 \quad (9)$$

The system of state co-state equations is defined by

$$\Delta\dot{h} = V_{tl}\sqrt{1+f_x^2}\sin(\Delta\psi - \psi_m) \quad (2)$$

$$\begin{cases} \frac{d\lambda_{\Delta\bar{\psi}}}{dt} = -\frac{\partial H}{\partial\Delta\bar{\psi}} = -\lambda_{\Delta h}\bar{V}_{tl}\cos\Delta\bar{\psi} \\ \frac{d\lambda_{\Delta h}}{dt} = -\frac{\partial H}{\partial h} = 0 \end{cases} \quad (10)$$

The optimal solution is determined by the expression $\frac{\partial H}{\partial n_z} = 0$ we get:

$$n_{zm} = -\frac{g}{\bar{V}_{tl}}\lambda_{\Delta\bar{\psi}} \quad (11)$$

From the Terminant function (7) we have the transformation boundary conditions:

$$\begin{cases} \lambda_{\Delta\overline{\psi}}\Big|_{t_f} = \dfrac{\partial G}{\partial \Delta\overline{\psi}}\Big|_{t_f} = \rho_1 \Delta\overline{\psi}\Big|_{t_f} \\ \lambda_{\Delta h}\Big|_{t_f} = \dfrac{\partial G}{\partial \Delta h}\Big|_{t_f} = \rho_2 \Delta h\Big|_{t_f} \end{cases} \quad (12)$$

In matrix form:

$$\begin{bmatrix} \lambda_{\Delta\overline{\psi}} \\ \lambda_{\Delta h} \end{bmatrix}_{t_f} = \begin{bmatrix} P^f \end{bmatrix} \begin{bmatrix} \Delta\overline{\psi} \\ \Delta h \end{bmatrix}_{t_f}; P^f = P^f = \begin{bmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{bmatrix} \quad (13)$$

System (11) is rewritten in matrix form:

$$\begin{bmatrix} \Delta\dot{\overline{\psi}} \\ \Delta\dot{h} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \overline{V}_{tl} & 0 \end{bmatrix} \begin{bmatrix} \Delta\overline{\psi} \\ \Delta h \end{bmatrix} + \begin{bmatrix} \dfrac{g}{\overline{V}_{tl}} \\ 0 \end{bmatrix} n_{zm} \quad (14)$$

In there : $\sin\Delta\overline{\psi} \approx \Delta\overline{\psi}$

$$\dot{x} = A^{(11)}x + Bu \quad (15)$$

with: $A^{(11)} = \begin{bmatrix} 0 & 0 \\ \overline{V}_{tl} & 0 \end{bmatrix}, B = \begin{bmatrix} \dfrac{g}{\overline{V}_{tl}} \\ 0 \end{bmatrix}, x = \begin{bmatrix} \Delta\overline{\psi} \\ \Delta h \end{bmatrix}, u = n_{zm}.$

From (14), it can be rewritten in matrix form as:

$$\begin{bmatrix} \dot{\lambda}_{\Delta\overline{\psi}} \\ \dot{\lambda}_{\Delta h} \end{bmatrix} = \begin{bmatrix} 0 & \overline{V}_{tl}\cos\Delta\overline{\psi} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{\Delta\overline{\psi}} \\ \lambda_{\Delta h} \end{bmatrix}$$

Or :

$$\dot{\lambda} = A^{(22)}\lambda \quad (16)$$

with: $A^{(22)} = \begin{bmatrix} 0 & \overline{V}_{tl}\cos\Delta\overline{\psi} \\ 0 & 0 \end{bmatrix}; \lambda = \begin{bmatrix} \lambda_{\Delta\overline{\psi}} \\ \lambda_{\Delta h} \end{bmatrix}$

From (13), (14) we have: $\dot{x} = A^{(11)}x - B\dfrac{g}{\overline{V}_{tl}}[1 \quad 0]\lambda$

$$\dot{x} = A^{(11)}x + A^{(12)}\lambda \quad (17)$$

with:

$$A^{(12)} = -\left(\frac{g}{\overline{V}_{tl}}\right)^2 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

From (18), we have the state equations at the terminal time:

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \end{bmatrix} = \begin{bmatrix} A^{(11)} & A^{(12)} \\ A^{(21)} & A^{(22)} \end{bmatrix} \begin{bmatrix} x \\ \lambda \end{bmatrix} \quad (18)$$

With: $A^{(21)} = [0]_{2\times2}$

From (18), we have the state equations at the terminal time:

$$\begin{bmatrix} x \\ \lambda \end{bmatrix}_{t_f} = \begin{bmatrix} I_{2\times2} \\ P^f \end{bmatrix} x_{t_f} \quad (19)$$

Proceeding with transformations similar to formulas (13)–(16), we obtain the guidance law defined by:

$$n_{zm} = -\frac{g}{\overline{V}_{tl}}[1 \quad 0]P^T(t)x \quad (20)$$

With P(t) obtained from the Riccati equation:

$$\dot{P} = \begin{bmatrix} \overline{V}_{tl}(\cos\Delta\overline{\psi}P_{12} - P_{21}).\left(\dfrac{g}{\overline{V}_{tl}}\right)^2 P_{11}^2 & -\overline{V}_{tl}P_{22} + \left(\dfrac{g}{\overline{V}_{tl}}\right)^2 P_{11}P_{12} \\ \overline{V}_{tl}\cos\Delta\overline{\psi}P_{22} + \left(\dfrac{g}{\overline{V}_{tl}}\right)^2 P_{21}P_{11} & \left(\dfrac{g}{\overline{V}_{tl}}\right)^2 P_{21}P_{12} \end{bmatrix}$$

Let: $k_1 = \dfrac{g}{\overline{V}_{tl}}P_{11}; k_2 = \dfrac{g}{\overline{V}_{tl}}P_{21}$ we obtain: $n_{zm} = -k_1\Delta\psi - k_2\Delta h$. Proceed to find an approximate solution by adding constraints on the system's performance criteria, following the same steps according to formulas (16)–(20), which yields the computational formulas for the coefficients:

$$k_1 = \frac{n_{zm-CP} + \dfrac{1}{gT_{CP}^2}\Delta h_{\max}}{\dfrac{1}{\overline{V}_{tl}T_{CP}}\Delta h_{\max} + \Delta\overline{\psi}_{\max}} \quad (21)$$

$$k_2 = \frac{n_{zm-CP} + \dfrac{\overline{V}_{tl}}{gT_{CP}}\Delta\overline{\psi}_{\max}}{\Delta h_{\max} + \overline{V}_{tl}T_{CP}\Delta\overline{\psi}_{\max}} \quad (22)$$

And the optimal guidance law with respect to angular and linear deviations, where the reference trajectory is the standard trajectory, is:

$$n_z = -\frac{k_1\Delta\overline{\psi} + k_2\Delta h}{\sqrt{1+f_x^2}} + \frac{V_{tl}}{g}\dot{\psi}_m \quad (23)$$

Thus, we obtain the guidance law (23) with coefficients determined by (21) and (22). In the guidance law (23), it can be seen that the first term is used to eliminate the trajectory deviation of the missile relative to the reference trajectory in terms of angular and linear errors, while the second term represents the required overload determined according to the reference trajectory.

If the reference trajectory is a straight line, we have the first derivative: $f_x^2 = 0$; $\dot{\psi}_m = f_{xx}\cos^2\psi_m V_t \cos\Delta\psi = 0$. It is evident that the guidance law (23) corresponds to the case where the reference trajectory is the standard trajectory.

III. SIMULATION RESULTS AND ANALYSIS

Simulation of standard trajectory guidance law test (23)

- Rocket velocity: $V_m = 1800 m/s$;
- Allowable overload: $n_{zmax} = 30$;
-Desired point coordinates M: $(x_m^*, z_m^*) = (70, 25)km$;

- Coefficient of optimal polynomial A1 = A2 = 0.46

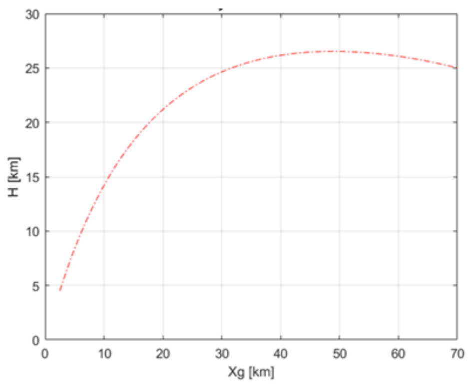

Figure 2. Missile trajectory

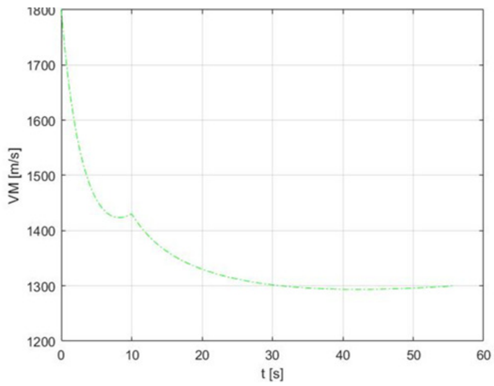

Figure 3. Missile velocity

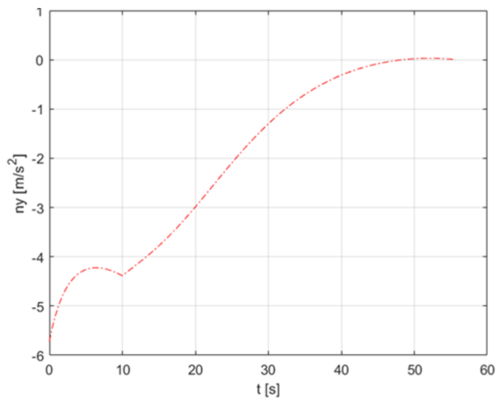

Figure 4. Missile required normal overload

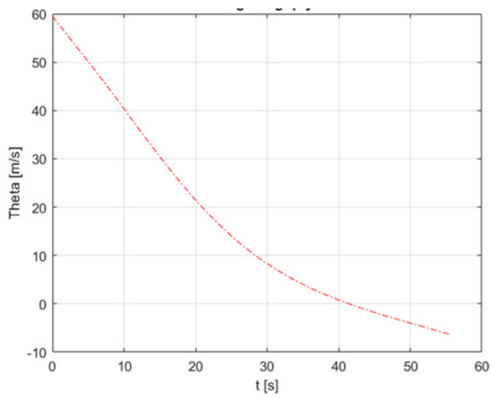

Figure 5. Orbital tilt angle

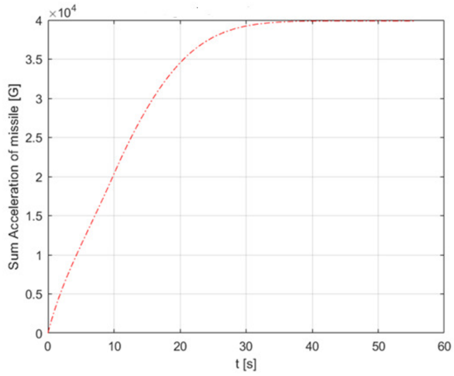

Figure 6. Control energy of the Conduction Method:

Comment: The obtained trajectory exhibits a convex shape, which represents the desired energy-optimal trajectory. The normal overload reaches its maximum at the initial stage and gradually decreases to zero at the predicted intercept point. The control energy consumption does not increase during the final stage, from 30 seconds to the end.

IV. CONCLUSION

This study proposed an optimal guidance law synthesis for surface-to-air missiles (SAMs) in the remote-control phase using the standard-trajectory method. By formulating the engagement as an optimal control problem and employing a reference standard trajectory, the derived laws ensure near-optimal performance while remaining suitable for real-time implementation.

The simulation results confirm that the obtained missile trajectory exhibits a convex shape, corresponding to the desired energy-optimal flight path during the remote-control phase. The normal overload reaches its peak value at the initial stage of the engagement and gradually decreases to zero as the missile approaches the predicted intercept point, ensuring a smooth and stable terminal approach. Furthermore, the control energy remains stable in the final stage (from 30 seconds to impact), indicating that the proposed guidance law effectively minimizes unnecessary control effort while maintaining high interception accuracy.

These characteristics demonstrate that the synthesized guidance law not only ensures energy-optimal performance but also provides favorable dynamic behavior throughout the engagement. The results validate the effectiveness of the standard-trajectory-based optimal control approach and highlight its potential for practical implementation in modern surface-to-air missile systems.

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