Synthesis of an Optimal Multi-Loop Range Coordinate Tracking Filter for Highly Maneuverable Targets

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Abstract:

This paper presents the synthesis of an optimal multi-loop range coordinate tracking filter for highly maneuverable targets. The single-loop range finder has weaknesses in tracking agile targets due to prolonged transient processes, non-linearity, decreased stability, and performance when increasing the static multiplicity. The proposed multi-loop range finder employs optimal filtering theory to improve target tracking quality and overcome the limitations of single-loop range finders. The range tracking system evaluates the tracked range, approach speed, acceleration, and target speed along the line-of-sight. Kalman filters are synthesized for the acceleration, controller, and tracked coordinate filters, ensuring accurate tracking, fast response, and high stability. Simulation results demonstrate the effectiveness of the proposed optimal multi-loop range tracker in tracking highly maneuverable targets with increased accuracy and robustness compared to conventional single-loop trackers

Keywords: **Range tracking, maneuverable targets, multi-loop range, Kalman filter, optimal filtering, target tracking**

I. INTRODUCTION

The single-loop range finder has weaknesses when tracking highly maneuverable targets. This is because in the single-loop range finder, the sensitive element, controller, and actuator are connected in series. To ensure high accuracy, additional integration stages are needed to evaluate the range and its derivatives. The target range is measured by the delay time of the reflected signal, while its derivatives are measured using the static multiplicity of the range finder. Thus, increasing the static multiplicity allows measurement of the derivative components. However, increasing the static multiplicity of the range finder above 2 leads to the following negative phenomena:

First, it prolongs the transient process when acquiring the target, greatly affecting the stability and performance of the range finder when increasing its static multiplicity.

Second, the range and speed values taken from the actuator increase the kinematic error and prolong the transient processes. When the target maneuvers in a complex way, higher order derivatives appear in the range finder that exceed the static multiplicity of the single-loop tracking systems, possibly causing loss of lock. For multi-loop range finders that evaluate the approach speed, based on optimal control and filtering algorithms, they can meet the requirements of noise suppression, high accuracy and ensure stable tracking, suitable for highly maneuverable targets.

The noise suppression capability of the multi-loop range finder is reflected in two aspects:

First, information about the range and approach speed to the target in the multi-loop tracker lies in the evaluation and controller output signals. More information is obtained, thus enhancing the tracking ability.

Second, the formation of an independent evaluation signal of the missile and target approach speed based on two different processing channels, the channel processing the reflected signal delay time and the Doppler frequency measurement channel. The presence of both independent approach speed evaluation channels will significantly improve the noise immunity of the tracker, especially when suppressing noise following the range and speed.

II. SYNTHESIS OF OPTIMAL MULTI-LOOP RANGE TRACKING SYSTEM

To acquire a maneuvering target, evaluation signals are needed for the tracked range $\hat{D}_{bs}$, approach speed $\hat{V}_{bs}$ and acceleration $\hat{j}_{bs}$, as well as the target speed along the line-of-sight $\hat{V}_{mt}$.

The tracking process model:

$$\dot{D}_{bs} = V_{bs}, D_{bs}(0) = D_{bs0} \quad (1)$$

$$\dot{V}_{bs} = j_{bs} = a_{mt} + a_r, V_{bs}(0) = V_{bs0} \quad (2)$$

$$\dot{a}_r = \xi_{ar}, a_r(0) = a_{r0} \quad (3)$$

$$\dot{a}_{mt} = -\alpha a_{mt} + \xi_{amt}, a_{mt}(0) = a_{mt0} \quad (4)$$

The controller model:

$$\dot{D}_{dk} = V_{dk}, D_{dk}(0) = D_{dk0} \quad (5)$$

$$\dot{V}_{dk} = b_v u_v + \xi_{dkv}, V_{dk}(0) = V_{dk0} \quad (6)$$

Where: a_r and a_{mt} are the missile's own acceleration and the target acceleration along the line-of-sight;

α is a coefficient characterizing the target maneuverability;

u_v is the efficiency coefficient of the control signal;

ξ_{ar}, ξ_{amt}, ξ_{dkv} are centered Gaussian noise with known one-sided power spectral densities G_{ar}, G_{amt}, G_{dkv} or known variances.

Using model (3) for the missile's own acceleration ar is simpler than model (4) for the target acceleration amt, since process (3) can be measured by an accelerometer. Therefore, the prediction error of process (3) can be compensated by observing this process. With the chosen state model (1)-(6), the measurements must satisfy the observability condition. The observability criterion indicates that to evaluate all components of the state vector, at least the lowest-order derivatives must be observed in each group of functionally-related coordinates. Therefore, to evaluate (1) and (6), at least D_{bs} and D_{dk} must be observed. Moreover, to separately evaluate a_{mt} and a_r, a_r needs to be observed. Information about D_{bs} and D_{dk} can be obtained from the discriminator and the controlled range sensor, while information about ar comes from the accelerometer.

The observation process model:

$$z_D = k_D(D_{bs} - D_{dk}) + \xi_{Dd}; \quad (7)$$

$$z_{Ddk} = k_{Ddk} D_{dk} + \xi_{Ddkd}; \quad (8)$$

$$z_{ar} = k_{ar} a_r + \xi_{ard}; \quad (9)$$

Where: z_D is the voltage at the output of the time discriminators; k_D is the gain of the discriminators; z_{Ddk} and z_{ar} are the output voltages of the controller and accelerometer; k_{Ddk} and k_{ar} are the gains of the meters; ξ_{Ddkd} and ξ_{ard} are centered white Gaussian noise at the meters with one-sided power spectral densities G_{dkd}, G_{ard} or known variances.

For the missile's own acceleration a_r, it can be measured by an accelerometer placed in the compartment, so its model is simpler than the model of the target acceleration a_{mt}. The target speed is not present in the phase coordinate model but can be fully evaluated based on the missile's own speed $\hat{V}_r$. Thus, the projection of the target speed onto the line-of-sight can be evaluated by algebraically summing the evaluation signals of the projections $\hat{V}_{bs}$ and $\hat{V}_r$.

Since the initial models (1)-(9) are linear, and the noise is white Gaussian, with a quadratic quality function, the optimal filter and optimal controller synthesis problems can be solved independently.

To solve these tasks for the prediction part (5)-(6) used to track process (1)-(2), with the presence of (7)-(8), the control signal u_v must be created:

$$u_v = K^D(\hat{D}_{bs} - \hat{D}_{dk}) + K^V(\hat{V}_{bs} - \hat{V}_{dk}) \tag{10}$$

K^D, K^V are the gains for the range and speed tracking errors.

The range filter is synthesized based on models (1)-(9) which define the general state vector:

$$x = \begin{bmatrix} D_{bs} & V_{bs} & a_r & a_{mt} & D_{dk} & V_{dk} \end{bmatrix}^T \tag{11}$$

And the measurement vector:

$$z = \begin{bmatrix} z_D & z_{Ddk} & z_{ar} \end{bmatrix}^T \tag{12}$$

The initial state vector (11) can be divided into three sub-vectors: $x_{bs} = \begin{bmatrix} D_{bs} & V_{bs} & a_{mt} \end{bmatrix}^T$, $x_{dk} = \begin{bmatrix} D_{dk} & V_{dk} \end{bmatrix}^T$, $x_a = a_r$ for each of which a separate filter can be synthesized. These include:

The acceleration filter:

$$\begin{aligned} &\dot{\hat{a}}_r = k_a \Delta z_a, \hat{a}_r(0) = z_{ar}(0)/k_{ar} \\ &\Delta z_a = z_{ar} - k_{ar}\hat{a}_r \end{aligned} \tag{13}$$

$k_a(k)$ is the time-varying Kalman filter gain for the acceleration filter.

The controller filter:

$$\begin{aligned} &\dot{\hat{D}}_{dk} = \hat{V}_{dk} + k_{dk1}\Delta z_{Ddk}, \hat{D}_{dk}(0) = z_{D.dk}(0)/k_{Ddk} \\ &\dot{\hat{V}}_{dk} = b_v u_v + k_{dk2}\Delta z_{Ddk}, \hat{V}_{dk}(0) = V_{dk0} \end{aligned} \tag{14}$$

k_{dk1} and k_{dk2} are the tracking error gains Kalman controller filter.

The tracked coordinate filter:

$$\begin{aligned} &\dot{\hat{D}}_{bs} = \hat{V}_{bs} + k_{bs1}\Delta z_{bs}, \hat{D}_{bs}(0) = D_{bs0} \\ &\dot{\hat{V}}_{bs} = \hat{a}_{mt} + \hat{a}_r + k_{bs2}\Delta z_{bs}, \hat{V}_{bs}(0) = V_{bs0} \\ &\dot{\hat{a}}_{mt} = -\alpha\hat{a}_{mt} + k_{bs3}\Delta z_{bs}, \hat{a}_{mt}(0) = 0 \\ &\Delta z_{bs} = z_{bs} - K_D\hat{D}_{bs} = z_D - K_D(\hat{D}_{bs} - \hat{D}_{dk}) \end{aligned} \tag{15}$$

k_{bsi} $(i = \overline{1,3})$ are the time-varying gains of the tracking error for the tracked coordinate filter.

III. SIMULATION AND EVALUATION

Simulation is done for the case where the target is incoming, moving with increasing acceleration, at the maximum maneuver frequency, so the tracking speed V_{bs}, target acceleration a_{mt} will be negative. The initial phase coordinates are given below:

Missile's own acceleration along line-of-sight: $a_r = 1(m/s^2)$;

Target acceleration along line-of-sight: $a_{mt} = -40(m/s^2)$;

Tracked range: $D_{bs} = 10000(m)$;

Tracked speed: $V_{bs} = -1800(m/s)$;

Missile speed: 1200 (m/s);

Target speed: 550 (m/s);

Target maneuver frequency: α = 0.12 (1/s);

Control signal efficiency coefficient: $b_v = 30 \div 80(m/v)$.

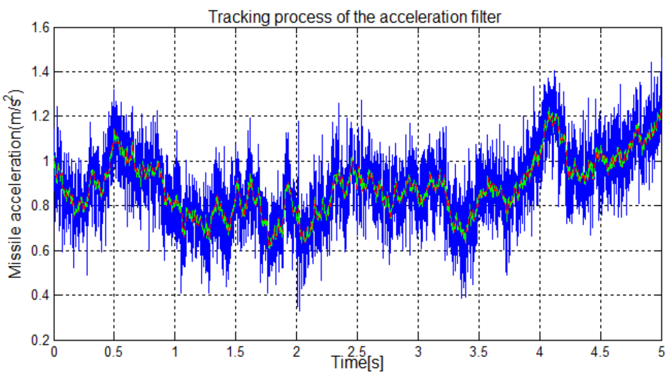

Figure 1. The tracking process of the acceleration filter

Since the missile's own acceleration along the line-of-sight is very small, we assume its initial value $a_r(0) = 1\ (\text{m/s}^2)$. Through the Kalman acceleration filter, the estimated $\hat{a}_r$ nearly coincides with the actual generated a_r. However, the directly measured missile acceleration z_{ar} from the accelerometer is quite noisy, so using this value directly in the system without filtering would cause errors. By using the Kalman filter, the noise from the accelerometer is eliminated, and the tracking estimate is quite good compared to the actual missile acceleration along the line-of-sight.

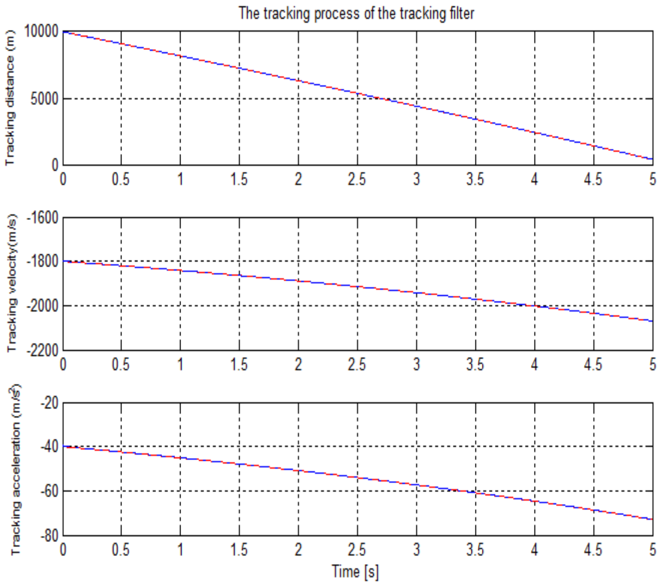

Figure 2. Tracking process of the range coordinate filter in the optimal multi-loop tracking system

From Figure 2, it can be seen that the estimated values of range, velocity, and target acceleration $(\hat{D}_{bs},\ \hat{V}_{bs},\ \hat{a}_{mt})$ closely follow their actual values $(D_{bs},\ V_{bs},\ a_{mt})$. When the target begins to maneuver with rapidly changing acceleration, the estimated values still maintain high stability and fast response. This demonstrates that the range coordinate filter, synthesized based on the optimal Kalman filtering algorithm, operates accurately and ensures precise target range tracking under strong maneuvering and noisy conditions.

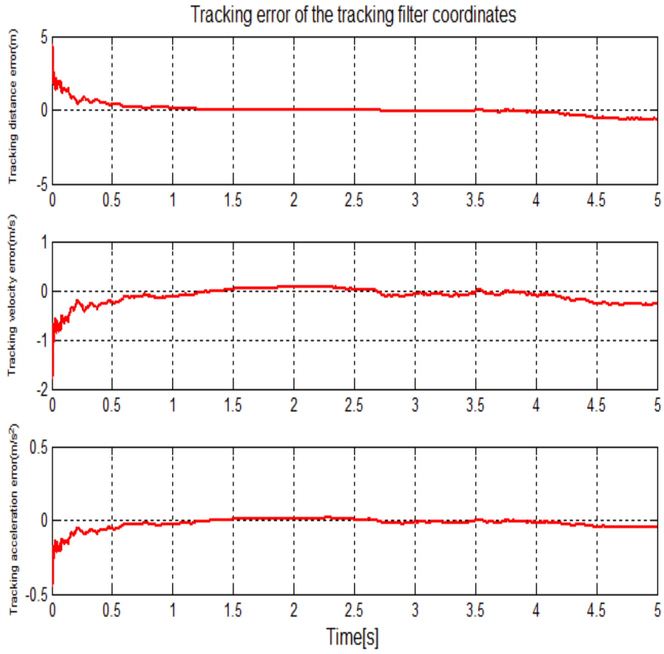

Figure 3. Range coordinate tracking errors in the optimal multi-loop tracking system

Figure 3 shows that the tracking errors between the actual and estimated phase coordinates $(D_{bs},\ V_{bs},\ a_{mt})$ quickly converge to zero after a short transient period. Although the errors slightly increase at the beginning due to the transient phase when the target changes its motion state, they are subsequently eliminated by the adaptive adjustment mechanism of the Kalman filter. This result confirms the stability, accuracy, and strong noise suppression capability of the range coordinate filter in the optimal multi-loop tracking system.

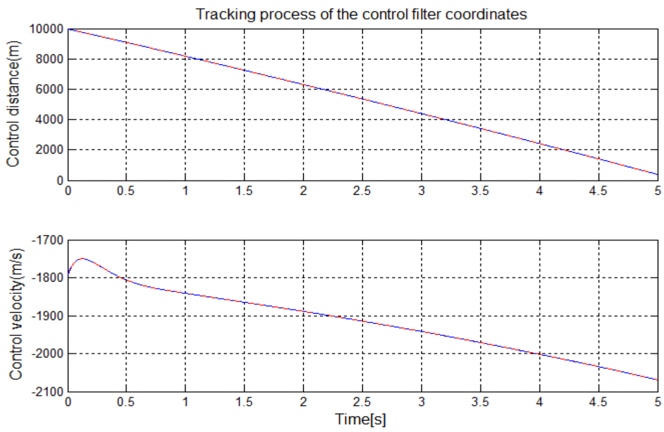

Figure 4. Tracking process of the controller in the optimal multi-loop tracking system

From Figure 4, it is observed that the estimated controller output signals ($\hat{D}_{dk}$, $\hat{V}_{dk}$) effectively follow the actual reference signals generated by the controller model. The estimated curves nearly coincide with the true signals, demonstrating that the controller operates stably and responds quickly to input variations. These results highlight the accuracy of the optimal controller in compensating range and velocity tracking errors within the multi-loop tracking system.

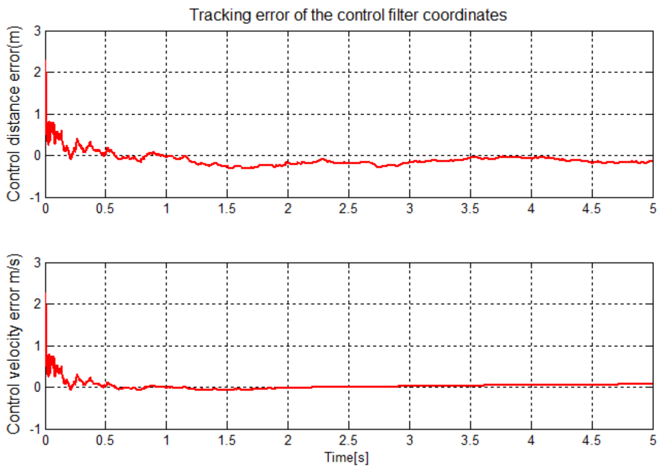

Figure 5. Controller tracking errors in the optimal multi-loop tracking system

Figure 5 illustrates the tracking errors between the actual control signals and their Kalman filter estimates. The errors decrease rapidly and converge to zero, indicating that the synthesized control and filtering algorithms are optimal. Although small oscillations appear at the beginning due to transient effects, the system quickly reaches a stable state. This confirms that the Kalman controller not only ensures high tracking accuracy but also enhances the stability margin of the entire optimal multi-loop range tracking system when engaging highly maneuverable targets.

IV. CONCLUSION

This paper has presented the synthesis and simulation of an optimal multi-loop range coordinate tracking filter for highly maneuverable targets. The proposed system effectively integrates three Kalman-based filters an acceleration filter, a range coordinate filter, and a controller filter forming an optimal multi-loop structure that enhances both the accuracy and dynamic stability of target range tracking.

The simulation results clearly demonstrate that the proposed multi-loop filtering architecture significantly improves the estimation precision of range, velocity, and acceleration compared with conventional single-loop tracking systems. The use of time-varying Kalman gains allows the filters to adapt dynamically to the target's maneuvering behavior, ensuring rapid convergence of estimation errors and strong robustness to measurement noise. The controller filter, synthesized within the same optimal framework, efficiently compensates for range and speed tracking deviations, thereby reducing the static multiplicity and improving the system's stability margins.

Moreover, the multi-loop design provides better observability of the target motion state, even under nonlinear and high-dynamic conditions. When the line-of-sight information is temporarily lost during rapid maneuvers, the system can still extrapolate the target's range position with acceptable accuracy, ensuring continuity of guidance information for missile control.

Overall, the proposed optimal multi-loop range coordinate tracking filter achieves fast response, high accuracy, and enhanced robustness against disturbances, making it well suited for next-generation radar tracking and missile guidance systems.

Future research will focus on extending this approach to adaptive multi-loop filters capable of handling non-Gaussian and time-correlated noise, integrating interacting multiple-model (IMM) estimation, and implementing real-time embedded verification for practical radar or seeker hardware.

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