 ISSN: 3107-6513

On Modelling and Optimal Solution of Electric Circuit Problems

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Abstract

Differential equations are of fundamental importance in both Physical Sciences and Engineering mathematics, because many physical laws and relations appear mathematically in the form of such equations and on this paper, a review and application of Some Methods for Solving Mathematical Models involved in some Science and Engineering problems, which consists of the First order linear differential equation is studied. Analytical approach is used in solving the equations particularly electric circuit problems.

Keywords: Ordinary Differential Equation, First Order Linear Differential Equation, Solution Circuit Problems and Quantity of Current.

Introduction

It gives the background to the study on which shows that in industries as well as commerce, the availability of fast and powerful computers have made it possible for mathematicians to serve a great range of problems previously difficult to solve because of their complexity.

Scientist often uses a mental model or theory to explain their observations, make predictions and then test their prediction by experiment. Mathematicians unlike other scientists do not deal directly with any physical objectives; their objects are ideas in mind, which have no physical existence. However most of the mathematical ideas are formed in the mind through idealization of physical objects. The step leading from the physical object to mathematical ideas is called modeling. (Bassier, 1971).

An electric circuit is a path or an interconnected group of paths (at least one which is closed), capable of carrying an electric current. Particularly, a circuit is any closed path with an electric network, a circuit containing a source of electromotive force E (a battery or a generator), a resistor , inductor L , a condenser (or capacitor) C and a switch all in series (Boyce et al., 2001; Dilwyn & Muke,1989; Galadanci, 2014). Resistor is an electrical component that limits or regulates the flow of electrical current in an electronic circuit. The resulting current is inversely proportional to the resistance. This is ohm's law, which states that the current (I)

 ISSN: 3107-6513

is equal to the voltage (V) divided by the resistance (R) that is I=V/R. Inductor is a passive electronic component that stores energy in the form of a magnetic field. In its simplest form, an inductor consists of a wire loop or coil. Inductance is directly proportional to the number of turns in the coil that is L. Capacitor is a small piece of electrical hardware that can hold electrical energy within a circuit or field. Also, capacitors are used in power supplies, amplifiers, Signal processors, oscillators and logic gate. Electromagnetic Force a fundamental force in nature, the electromagnetic force acts between charge particles and is the combination of all electrical and magnetic forces. Example, the electromagnetic force holds atoms together in molecules, causes friction and attracts iron to a magnet.

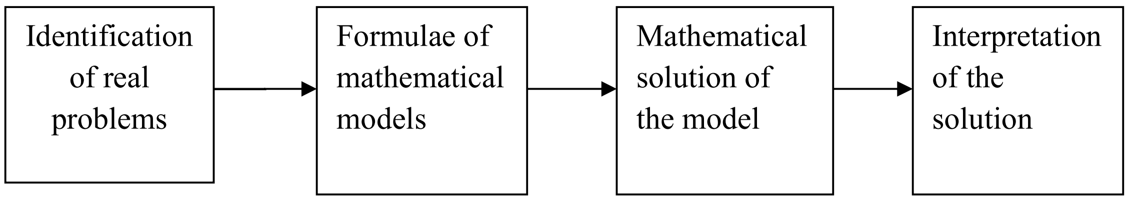
Models can be defined as simplified representation of certain aspect of life system. This may be physical, biological, and economical and so on. A model specifies problems, reduces complication and lead to easy solution of problems. This may include

(i) Deterministic models and
(ii) Stochastic models

A mathematical deterministic model is a representation $y = f(x)$ that allows you to make predictions of y based on x. The model is used like: when x = 3 then predict that $y = f(3)$.

In deterministic models, everything is certain while stochastic models are the one that corporate probability that involves number of activities. This type of model is used for situation where random effect plays an important role in problem under investigation.

Mathematical models can be represented by means of flow chart. A flow chart is a diagram representing the general steps involved toward solving problems. The flow chart for a typical problem is shown below**:**



An Ordinary Differential Equation (ODE) is defined as a differential equation that contains only ordinary derivatives of one or more unknown functions with respect to a single independent variable. ODEs can be classified as linear or nonlinear and have applications in various fields such as biology, physics, and

ISSN: 3107-6513

engineering. An example of solving an ODE is provided, along with its applications in modeling phenomena like electricity movement and disease growth.

This paper, studied the application of first order linear ordinary differential equations in some aspect of engineering particularly in solving electrical circuits problems on which determines the optimal quantity of current that pass through the circuit. The work is to review and apply some existing methods for Solving Mathematical Models involved in engineering problems particularly Electric circuit. The problems that will be considered and the models arising from the problems that which are not newly formulated or modeled from a real life situation or rather sourced from the workshop as a result of an experiment, but alternative way to solve such problems.

THE METHOD FOR FINDING THE OPTIMAL SOLUTION OF FIRST ORDER EQUATIONS

In this section, the methods for finding the optimal solution of first order linear ordinary differential equations are presented. In each method, an example is given to demonstrate how to obtain the solution to such equations.

- **Equation with Separable Variable**

A first order differential equation is said to be variable separable if it can be written in the form:-

$$\frac{dy}{dx} = \frac{g(x)}{f(y)} \qquad 1$$

$$f(y)dy = g(x)dx$$

Then we integrate both sides to obtain its solution of the differential equation (1) after separating and integrating both sides.

Examples: Find the solution to the following differential equations

(a) $\frac{dy}{dx} = \frac{y-1}{x+1}$

(b) $(1+x^2)\frac{dy}{dx} = 2xy$

Solutions

$$\frac{dy}{dx}=\frac{y-1}{x+1}$$

The variable already are separable, so

$$\frac{dy}{dx}=\frac{y-1}{x+1}$$

$$\int\frac{dy}{y-1}=\int\frac{dx}{x+1}$$

$$\ln(y-1)=\ln(x+1)+\ln A$$

$$\ln(y-1)-\ln(x+1)=\ln A$$

By law of logarithm, we have

$ln\frac{(y-1)}{(x+1)}=\ln A$

- **Integrating Factor**

An equation of the form

$$\frac{dy}{dx}+Py=Q \qquad (2)$$

Is a linear differential equation of first order, where P and Q are functions of x.

Multiplying both sides of equation (2) by the integrating factor e^{pdx} we get:

$$\left(\frac{dy}{dx}+py\right)e^{\int pdx}=Qe^{\int pdx}+C$$

Example: Solve the equation $\frac{dy}{dx}+\frac{3y}{x}=x^2$

Solution

$\frac{dy}{dx}+\frac{3y}{x}=x^2$ 3

Comparing with equation (2) above gives

$P=\frac{3}{x}, Q= x^2$

So,

The integrating factor is

$e^{\int pdx} = e^{\int \frac{3}{x}dx} = e^{3inx} = e^{ln\ ^3} = x^3$

Now, multiplying both sides of equation (3) by the integrating factor, we obtain

$\frac{d(yx^3)}{dx} = x^5$ 4

Integrating both sides

$yx^3 = \frac{x^6}{6} + C$

Multiply (3) through by x^{-3}

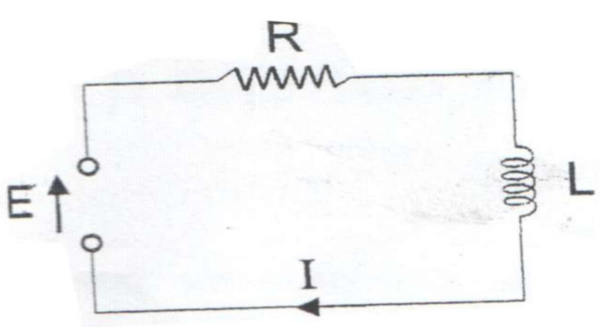
Therefore, $Y = \frac{x^3}{6} + Cx^{-3}$

APPLICATION ON FIRST ORDER LINEAR ORDINARY DIFFERENTIAL EQUATION OF ELECTRIC CIRCUIT

Problem 0.: In a model of electric circuit below. The figure shows a circuit containing an electromotive force, a capacitor with a capacitance of C farads (F) and a resistor of R ohms (Ω). A review shows that the voltage drop across the capacitor is $\frac{Q}{C}$, where Q is the charge (in coulombs), so in this case Kirchhoff's law gives:

$RI + \frac{Q}{C} = E(t)$

But $I = \frac{dQ}{dt}$ we have,



 ISSN: 3107-6513

Problem 1: A circuit containing an electromotive force, a capacitor with a capacitance of C farads (F) and a resistor of R ohms (Ω). Suppose the resistance is 4Ω, the capacitance $0.05F$, a battery gives a constant voltage of $60V$ and the initial charge is Q(0)=0.

(a) Find the charge and current at time t.

(b) What is the charge at t=2s?

Solution

Given that $RI + \frac{Q}{C} = E(t)$ where R=4, C=0.05 E (t)=60

$$4\frac{dQ}{dt} + \frac{1}{0.05}Q = 60$$

$$\Longrightarrow \frac{dQ}{dt} + 8Q = 15$$

Applying the Integrating Factor

$$\varphi = e^{\int P dt} = e^{8\int dt} = e^{8t}$$

Multiplying through by e^{8t}

$$e^{8t}\frac{dQ}{dt} + e^{8t}8Q = 15e^{8t}$$

$$\Longrightarrow \frac{d}{dt}(e^{8t}Q) = 15e^{8t}$$

Integrate both sides

$$e^{8t}Q = 15\int e^{8t}\, dt$$

$$e^{8t}Q = 15\left(\frac{e^{8t}}{8}\right) + C$$

Multiply through by e^{-8t}

$$Q = 1.875 + Ce^{-8t}$$

$$Q(0) = 1.875 + C \;\; \therefore C = -1.875$$

$Q(t) = 1.875 - 1.875e^{-8t}$

$Q(t) = 1.875(1 - e^{-8t})$

At time t=2

$Q(2) = 1.875(1 - e^{-8(2)})$

= 1.875 Columbs

Problem 2:

A generator supplies a voltage of (t)=40sin60 volts, the inductance is 1H, the resistance is 20Ω and I(0)=1. Find;

(a) Find I(t).

(b) Find the current after 0.1s.

Solution

Given that;

$L\frac{dI}{dt} + RI = E(t)$, where L = 1, R = 20, I = 1 and E (t) = 40sin60.

$\therefore \frac{dI}{dt} + 20I = 40sin60t$

Applying Integrating Factor;

$\varphi = e^{\int Pdt} = e^{20\int dt} = e^{20t}$

Multiplying through by e^{20t}

$e^{20t}\frac{dI}{dt} + e^{20t}20I = e^{20}\ 40sin60t$

$\Longrightarrow \frac{d}{dt}(e^{20}\ I) = e^{20}\ 40sin60t$

Therefore, $I = \frac{3e^{20}\ sin60t - 9e^{20t}cos6}{15} + Ce^{-20t}$

At initial point t=0

$I(0) = \frac{-9}{15} + C = 0$

$then, C = \frac{9}{15}$

a. At time t

$I(t) = \frac{3e^{20} \ sin60t - 9 \ ^{20} \ cos6}{15} + \frac{9}{15}e^{-20t}$

b. The current after 0.1s

$I(0.1) = \frac{3e^{20(0.1)}sin60(0.1) - 9e^{20(0.1)}cos60(0.1)}{15} + \frac{9}{15}e^{-20(0.1)}$

= 0 .4946 Columb.

RESULT AND DISCUSSION

In this study, we conclude that first order linear differential equations which are the simplest forms of ordinary differential equations, play an important role indifferent fields of Science and Engineering when formulated from a real or physical situation. Here, the application is studied in the problems of electrical circuits that provided the optimal solution.

CONCLUSION AND RECOMMENDATION

Based on the problems application and method discussed, it is recommended that applying first order linear differential equations in solving problems related to the electric circuits is proper and easier than the direct method. The analytic approach used in solving the equations confirmed that the solving electric circuits using first order linear ordinary differential equations gives accurate, efficient and reliable result. However, complex problems need higher order differential equations, which are nonlinear and have entirely different approach in finding their solutions. Therefore, the application is of significance and great need.

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